Annex

Hu's Moment Invariants

Two-dimensional $(p + q)$ the order moments are defined as follows:

$$m_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q f(x,y)dxdy \qquad (1)$$

$$p, q = 0,1,2,\ldots..$$

If the image function $f(x,y)$ is a piecewise continuous bounded function, the moments of all orders exist and the moment sequence $\{m_{pq}\}$ is uniquely determined by $f(x,y)$; correspondingly, $f(x,y)$ is also uniquely determined by the moment sequence $\{m_{pq}\}$. One should note that the moments in (1) may be invariant when $f(x,y)$ changes by translating, rotating or scaling. The invariant features can be achieved using central moments, which are defined as follows:

$$\mu_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - \bar{x})^p (x - \bar{y})^q f(x,y)dxdy \qquad (2)$$

$$p, q = 0,1,2,\ldots..$$

Where

$$\bar{x} = \frac{m_{10}}{m_{00}} \qquad (3) \qquad\qquad \bar{y} = \frac{m_{01}}{m_{00}} \qquad (3)$$

The pixel points $(x\ ,y)$ are the centroid of the image $f(x,y)$. The centroid moments μ pq computed using the centroid of the image $f(x,y)$ are equivalent to the mpq whose center has been shifted to the centroid of the image. Therefore, the central moments are invariant to image translations. Scale invariance can be obtained by normalization. The normalized central moments are defined as follows:

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}}, \gamma = \frac{p+q+2}{2}, p + q = 2,3, \ldots. \qquad (4)$$

Based on normalized central moments, Hu introduced seven moment invariants:

$$\phi_1 = \eta_{20} + \eta_{02} \qquad (5)$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \qquad (6)$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \mu_{03})^2 \qquad (7)$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \mu_{03})^2 \qquad (8)$$

$$\phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + 3(3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \tag{9}$$

$$\phi_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] - 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \tag{10}$$

$$\phi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] - (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \tag{11}$$

The seven moment invariants are useful properties that are unchanged under image scaling, translation and rotation.